Chapter 5
Multiplying with variables

Multiply the numbers and write the letters

\[
\begin{align*}
3(2x) & & 2x(3y) & & 4x(3) & & 3x(-2y) & & 5x(-2y-3x) \\
6x & & 6xy & & 12x & & 6xy & & 30x-15x \\
\end{align*}
\]

\[
\begin{align*}
5r & & (-2x-3y) & & -3x(-2y) & & 4x(3) & & 4x(3)-3x(-2y) \\
5r & & 5r & & 6xy & & 12x & & 12x-(-6xy) \\
& & 6xy & & 12x & & 12x+6xy \\
\end{align*}
\]

DISTRIBUTING

\[
\begin{align*}
3(x+2) & & 5(2x-1) & & -3(x+2) & & -3(3x-2) & & (5r-6x(-2)) \\
3x+6 & & 10x-5 & & -3x-6 & & -15x+6 & & -10r+12 \\
\end{align*}
\]

Expressions

Expressions do not have an equal sign.
Expressions are like your hair sometimes there too long and you need to cut them down to size. You just simplify them.

Add or Subtract (Must have like terms)

\[1 + 3 \rightarrow 4\]

The number in front tells you how many you have.
So, one apple plus three apples is four apples.

Remember this—the + means you can only count or remove your like terms, that is all.
YOU CAN NOT CANCEL, OR USE EXPONENT RULES.

A) \(5 + 3 = 8\) monkeys

B) \(4x + 7x = 11x\)

C) \(3 + 2 + 2 = 5\) monkeys

D) \(8x - 5x^2 + 6x^2 = -5+6 \rightarrow 8x + x^2\)
E) \(3x - 2x + y\)

\[
\begin{align*}
1x + 1y \\
-x + y
\end{align*}
\]

G) \(\frac{1}{6} + \frac{2}{3}

\[
\frac{3}{5}x
\]

F) \(12a - 4b - 8A - 2a\)

The sign belongs to the letter, so for the little a’s we have 12 + 4 = 16 - 2a

for the capital A’s we have 4A - 8A

This gives us a simplified answer of \(16 - 9A\)

1) \(-m - 4n - 8m + 4n\)

\[-9m + 0 \quad 6A + 5\]

\[-9m\]

2) \(9AB - 3AB\)

\[6A + 5\]

\[-7x - 8y + 4\]

3) \(\frac{3x - 6y - 10x + 4}{2}\)

4) \(\frac{2y - 4y}{9}\)

\[\frac{\frac{2}{3} - \frac{4}{9}}{2}\]

\[\frac{\frac{6}{9} - \frac{4}{9}}{2}\]

\[= \frac{2}{9}y\]

5) \(\frac{3x^2 + 5}{12}x^2\)

\[\frac{3}{8} + \frac{5}{12}x^2\]

\[\frac{3}{8}x - \frac{5}{12}x^2\]

6) \(\frac{0.25x - 3.2y - 9x}{0}\)

\[\frac{2.5}{75}x - \frac{3.2}{9}y\]

Expression #1

\[\frac{3(x - 2) - 4(-5x + 1)}{1}\]

It’s to long, cut it down to size.

\[\frac{3x - 6 + 20x - 4}{3x + 20x - 6 - 4} \quad \frac{23x - 10}{23x - 10}\]

1) Distribute

2) Combine like terms

Try:

\[-3 + (4x - 2) - [2 + 3x]\]

\[-3 + 4x - 2 - 2 - 3x\]

\[4x - 3x - 3 - 2 - 2\]

\[1x - 7\]

\[3f \left(t - 8\right)\]

\[3t - 16 + 8\]

\[2t + 5\]

\[4 + (t - 8)\]

\[4 + 16 - 8\]

\[1t - 4\]
Polynomials ——- a sum or difference of terms where the variables have whole # exponents

\[3x^2 + 2x^{-1}\] not a polynomial
\[4x^2 - 3x\] polynomial
\[\frac{1}{2}x - 3\] polynomial
\[\frac{1}{x^2} - x\] not a polynomial

<table>
<thead>
<tr>
<th>Monomial</th>
<th>Binomial</th>
<th>Trinomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>one term</td>
<td>two terms</td>
<td>three terms</td>
</tr>
<tr>
<td>[3x]</td>
<td>[3x^2 + 4]</td>
<td>[7x^3 + 4x^2 - 2]</td>
</tr>
<tr>
<td>[7x^5]</td>
<td>[7x + 5x^7]</td>
<td></td>
</tr>
</tbody>
</table>

Degree of a Polynomial — largest exponent

\[\frac{3x^2}{2x^{-1}}\] degree 3
\[4x^2 - 3x\] degree 2
\[\frac{1}{2}x - 3\] degree 1
\[\frac{1}{x^2} - x\]

Identify the following polynomials:

\[3x^2 + 2x + 4\] degree 2 trinomial
\[4x^3 + 3x^2 - 2\] polynomial degree 3
\[\frac{1}{2}x - 3\] binomial degree 1
\[\frac{1}{x^2} - x\] not a polynomial
Adding and Subtracting Polynomials

The number in front tells you how many you have.
So, one apple plus three apples is four apples.

Remember this—the + means you can only count or remove your like terms, that is all.
YOU CAN NOT CANCEL, DISTRIBUTE EXPONENTS.

Finish these (only add up the exact like terms, say them as you go):

A) \(5 + 3\)

B) \(4x + 7x\)

C) \(3 + 2 + 2\)

D) \(8x - 5x^2 + 6x^2\)

\[
\left(7x - 3x^2 + 5x - 2\right) + \left(4x^3 + 5x^2 - 2\right)
\]

\[
7x^3 + 3x^2 + 10x - 4
\]

\[
\left(-4x^3 - 5x^2 + 2\right) - \left(4x^3 + 5x^2 - 2\right)
\]

\[
-4x^3 - 5x^2 + 2
\]

\[
7x^5 - 5x^3 - 8x^2 + 5x
\]
\[
\frac{6x^3 + 8x - 1}{12x^3 - 3x^2 + 6x + 1} + \frac{5x^3 + 2}{8x^5 - 6x - 5x^3 - 2} + 5x^3 - 2
\]

\[
\left(x^5 - \frac{1}{2}x^3 - 2\right) + \frac{1}{2}x^3 - 2
\]

\[
\frac{-6x^3 - 0.335x + 3.5}{-6x^3 - 0.335x + 7.5}
\]

\[
\frac{-6x^3 - 1.03x^2 - 0.385x + 3.3}{-6x^3 - 1.03x^2 - 0.385x + 3.3}
\]

\[
\left(3x^2 - 2\right) + 4 - 5
\]

\[
\left(4y^3 + 5x - 2\right) - \left(3y^3 x - 3x^2 + 5x - 2\right)
\]

\[
7y^5 - 5y^3 x^3 - 3x^2 y^2 + 5x - 2
\]

\[
7y^5 - 5y^3 x^3 - 5x^2 y - 3x y^2 + 5x
\]
**Multiplying Polynomials**

1. Expand the following as above.

   a) \( x^5 \)  
   b) \( x^3 \cdot x^7 \)  
   c) \( x^3 \cdot x^4 \)  
   d) \( x^2 \cdot y^5 \cdot y \)

   - How many x's are you multiplying in each problem?
     - a) 5  
     - b) 10  
     - c) 7  
     - d) 5

   - What operation is happening between the exponents of the same bases?
     - Addition

2. Expand the following as above.

   a) \( (x^2)^4 \)  
   b) \( (x^2 \cdot y)^4 \)  
   c) \( (x^3)^3 \)  

   - How many x's are you multiplying in each problem?
     - a) \( x^8 \)  
     - b) \( x^8 \cdot y^{12} \)  
     - c) \( 16 \cdot x^{12} \)

---

Expand \( 2 \cdot (3x^2 + 3) \cdot 2 + 3 \)  

Expand \( (x^2)(x^2)(x^2) \)
1) \( r^2 r^2 = r^6 \)
2) \( x^1 x^4 = x^5 \)
3) \( 5r^2 r^2 = 5^2 \)
4) \( -6x(4x) = -24x^5 \)
5) \( 6y^3 y^4 y^3 = 6y^{10} \)
6) \( -x^4 \)
7) \( -2r(-4r) = 8r^2 \)
8) \( 3x^3 \cdot y^2 = x^{2} y^4 \)
9) \( b^6 \cdot c^2 \cdot b^1 \cdot c^3 = b^7 \cdot c^5 \)
10) \( -3x^3 (-5r^2) = 15x^3 r^2 \)

Now all together:

\[
\begin{align*}
\left(2a^2\right)^2 \cdot \left(6a^3\right) & = 14a^9 - 30a^9 \\
& = -16a^9
\end{align*}
\]
### Multiplying Polynomials

<table>
<thead>
<tr>
<th>Monomial • (polynomial)</th>
<th>Binomial • Binomial</th>
<th>Binomial • Trinomial</th>
</tr>
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<tbody>
<tr>
<td>$3x^2(2x^3)$</td>
<td>$3x^7(2x)$</td>
<td>$(4x + 3)(3x^2 + 2x + 5)$</td>
</tr>
<tr>
<td>$-5x^2(x^3)$</td>
<td>$-5x^{10} + 5x^9$</td>
<td>$-2x^2 + 3x - 1$</td>
</tr>
<tr>
<td>$x^4(x^3)$</td>
<td>$x^{13}$</td>
<td>$-2x^2 + 3x + 1$</td>
</tr>
<tr>
<td>$-x(-x^3)$</td>
<td>$x^3$</td>
<td>$-2x^3 + 9x^2 - 10x + 3$</td>
</tr>
<tr>
<td>$6x^5$</td>
<td>$-5x^9$</td>
<td>$3x^2 + 6x + 15$</td>
</tr>
<tr>
<td>$-3x^9$</td>
<td>$2x^2 + 9x + 45$</td>
<td>$x^2 + 2xy - 15y^2$</td>
</tr>
<tr>
<td>$-1x^{10} - 2x^4 + 3x^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5x^6 + 6x^2 + 15$</td>
<td></td>
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<tr>
<td>$-3 - 6x + 45$</td>
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<td>$2x^2 + 9x + 45$</td>
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<tr>
<td>$6x^2 + 26x + 15$</td>
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<tr>
<td>$-1x^2 + 5x + 5y$</td>
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<td>$-4x + 20x$</td>
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<td>$-3 + 6x + 15$</td>
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<td>$12x^3 + 17x^2 + 6x + 15$</td>
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SCIENTIFIC NOTATION

3,654,000,000,000,000,000,000,000,000,000,000
0.0000000000123

1) 3.654 \times 10^{23}
2) 2.58 \times 10^{19}
3) 5.89
4) 6.57 \times 10^{-7}
5) 6.5 \times 10^{-10}
6) 5.89 \times 10^{-7}
7) 5.68 \times 10^1
8) 9,999 \times 10^4
9) 1,235,555 \times 10^2
10) 5.065 \times 10^{-6}

Scientific Notation to Decimal form

1) 3.15 \times 10^6 \rightarrow 3,150,000
2) 3.15 \times 10^{-6} \rightarrow 0.00000315
3) 9.35 \times 10^{-6} \rightarrow 0.00000935
4) 1.789 \times 10^{-6} \rightarrow 0.0000001789
5) 9 \times 10^{-6} \rightarrow 0.000009
6) 3 \times 10^{-6} \rightarrow 0.000003
7) 9,655,555 \times 10^{-6}
Phrases

The difference of 4 and 3 \( (4 - 3) \)  
The sum of 4 and 3 \( (4 + 3) \)  
The product of 4 and 3 \( (4 \cdot 3) \)  
The quotient of 4 and 3 \( (4 \div 3) \)

A number \( x \) a variable x, n, t, etc.

Example: four times the sum of five and a number.
\[ 4 \cdot \left( 5 + x \right) \rightarrow 4(5 + x) \]

TWO SPECIAL CASES

A number subtracted from 3
\[ X - 3 \]
\[ 3 - X \]

7 less than a number
\[ 7 - X \]
\[ X - 7 \]

Translate each phrase.

1. The product of 3 and 4 \( (3 \cdot 4) \)
2. The sum of 7 and 8 \( (7 + 8) \)
3. 4 subtracted from 8 \( 8 - 4 \)
4. The quotient of 6 and 3 \( (6 \div 3) \)
5. 8 times the sum of 4 and 3 \( 8 \cdot (4 + 3) \)
6. The sum of 4 times 2 and 5 times 3 \( (4 \cdot 2) + (5 \cdot 3) \)
7. 4 minus the sum of 5 and 2 \( 4 - (5 + 2) \)
8. The difference of 6 and 4 \( (6 - 4) \)
9. 7 less than a number \( x - 7 \)
10. 7 less a number \( 7 - x \)
11. twice the price p \( 2p \)
12. half of a number \( \frac{1}{2}x \)
13. 7 divided by a number \( \frac{7}{x} \)
14. exceeds x by 7 \( x + 7 \)
15. 5 increased by \( t \) \( 5 + t \)
16. 6 minus x \( 6 - x \)
17. The product of 3 and the sum of a number and 4 \( 3 \cdot (x + 4) \)
Translate the following and simplify if possible.

4. The quotient of 6 and 3  
5. 8 times the sum of 4 and 3  
6. The sum of 4 times 2 and 5 times a 

7. 4 minus the sum of 5 and 2.  
8. The difference of 6 and 4  

9) The sum of 4 and the difference of 3 and a number.  

10) The product of a number and the sum of 6 and the same number.  

11) half of the quotient of 3 and a number  
12) The quotient of x and the sum of x and 3.  

13) The difference of 7 and the product of 4 and a number  

14) The difference of the product of 3 and a number and the sum of 5 and the same number.  

15) The sum of 5 and the square of t.  

16) one third of the square of x.  

17) 4 less than the cube of y.
Expressing one object in terms of another using a variable.

1. If the length is 4 ft more than the width. Express the length and width using one variable.
   \[ \text{Length} = 4 + w \quad \text{width} = w \]

2. There are 4 times as many apples as pears in a bowl. Express the number of apples and pears using one variable.
   \[ \text{Number of apples} = 4p \quad \text{Number of pears} = p \]

3. The base of a triangle is four less than the height. Express the height and base using one variable.
   \[ \text{base} = h - 4 \quad \text{height} = h \]

4. One cyclist rides six miles per hour faster than another cyclist. Express the speed of the faster cyclist in terms of the speed of the slower cyclist.
   \[ \text{Speed of faster cyclist} = s + 6 \quad \text{speed of slower cyclist} = s \]

5. The planet Saturn has 7 more moons than Jupiter. Express the number of moons Saturn has in terms of the number of moons Jupiter has.
   \[ \text{# of moons of Saturn} = 7 + J \quad \text{# of moons of Jupiter} = J \]

6. The sale price of a suit is 3/4ths of the original price. Express the sale price of the suit in terms of the original price.
   \[ \text{sale price} = \frac{3}{4} \times X \quad \text{original price} = X \]

7. A bank contains 12 coins made up of dimes and nickels. Express the number of nickels in terms of the number of dimes.
   \[ \text{# of nickels} = 12 - x \quad \text{# of dimes} = x \]